

Nucleon electromagnetic form factors from lattice QCD

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Abstract. It is imperative that lattice QCD serve to develop our understanding of hadron structure and, where possible, to guide the interpretation of experimental data. There is now a great deal of effort directed at the calculation of the electroweak form factors of the nucleon, where for example, measurements at Jefferson Laboratory have recently revealed surprising behaviour in the ratio G_E/G_M . While, for the present, calculations within the framework of lattice QCD are limited to relatively large quark mass, there has been considerable progress in our understanding of how to extrapolate to the chiral limit. Here we report the results of the application of these techniques to the most recent form factor data from the QCDSF Collaboration. The level of agreement with all of the form factors, for Q^2 below 1 GeV^2 , is already impressive.

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1 Introduction

The electromagnetic form factors of the nucleon provide a fundamental constraint for any theoretical description of the structure of the nucleon [1]. Even though they have been studied experimentally for more than 50 years, recent experiments at Jefferson Lab [2] have revealed surprising new behaviour in the ratio G_E/G_M for the proton. In addition, there are a range of other new results on the neutron electric and magnetic form factors [3] – this is an exciting and rapidly developing field.

In this context it is vital that lattice QCD, our only rigorous method of solving non-perturbative QCD, be used to inform our understanding of this data and the various models of hadron structure that are used to describe it. There was a relatively long hiatus in calculations of nucleon form factors in lattice QCD after the pioneering work of Leinweber and collaborators in the early 90's [4, 5, 6]. However, the activity has intensified in the last few years [7, 8, 9]. In spite of this activity, limitations in computer speed mean that we are currently limited to lattice simulations at quark masses a factor 5-10 higher than the physical values and these calculations are currently made in quenched approximation (QQCD).

If one is to compare these state of the art simulations with experiment, it is necessary to make an extrapolation as a function of quark mass to the physical region [10, 11]. This extrapolation is made non-trivial by the non-analytic behaviour as a function of m_q which follows from the fact that chiral symmetry is dynamically broken in

QCD. Until recently the absence of data has meant that efforts at chiral extrapolation of form factors has been focussed on baryon magnetic moments [12, 13, 14, 15] and charge radii [16, 17]. The focus on non-analyticity [18] of hadron electromagnetic properties has inspired considerable investigation of the formal constraints in both full QCD and QQCD [19, 20, 21, 22, 23].

The practical issue is then how to incorporate these formal chiral constraints into a practical chiral extrapolation, given that the radius of convergence of the formal expansion dictated by chiral perturbation theory is rather small [24]. In the case of the nucleon mass, where there is extremely accurate data in full QCD from the CP-PACS Collaboration [25], one can formally demonstrate the model independence of the extrapolation procedure [24, 26, 27]. For the magnetic moments the data is only now improving to the point where the model independence of the choice of finite range regulator [28] can be examined [12]. In the case of the form factors this is not yet possible.

The procedure employed in this paper is to draw on the earlier phenomenological experience with chiral extrapolations of the magnetic moments and charge radii, taking simple phenomenological functional forms which build in the correct non-analytic behaviour and any other asymptotic constraints that are known [29]. For the present the accuracy of the lattice data is not such that it could discriminate between different functional forms for the Q^2 -dependence of the nucleon form factors. In particular, it cannot yet address the JLab issue of whether G_E/G_M decreases as Q^2 increases. We therefore parametrize sep-

arately the isoscalar and isovector lattice data, at a given value of m_q , or m_π , as a dipole and then extrapolate the dipole mass as a function of m_π , building in the appropriate chiral constraints. Clearly this simple approach can be systematically improved as the lattice simulations become capable of making a better discrimination between possible functional forms. For the present we shall see that our relatively simple approach already produces quite impressive results.

2 Extrapolations

It is well known [1] that the Q^2 -dependence of the nucleon Sachs electromagnetic form factors (with the exception of the neutron electric form factor G_E^n) are described in first approximation by a dipole form

$$G(Q^2) = \frac{G(0)}{(1 + Q^2/\Lambda^2)^2}. \quad (1)$$

Here Λ is the dipole mass and the charge form factor of the proton satisfies $G_E^p(0) = 1$, while $G_M^p(0) = \mu_p$ and $G_M^n(0) = \mu_n$ are the proton and neutron magnetic moments. As explained in the Introduction, we will use this phenomenological fact to construct a simple but effective extrapolation formula.

To isolate the chiral behaviour of the form factors we rearrange them into isovector and isoscalar combinations:

$$G^v = G^p - G^n \quad (2)$$

and

$$G^s = G^p + G^n, \quad (3)$$

respectively. These also display a dipole-like Q^2 -dependence but with different dipole masses and magnetic moments.

To extrapolate lattice QCD results for the electromagnetic form factors from the large pion masses at which they are calculated to the physical regime, we extract dipole masses and magnetic moments from the lattice data and then extrapolate these as a function of m_π .

Following [13,14], one can use a Padé approximant which builds in both the correct chiral non-analytic behaviour as $m_\pi \rightarrow 0$ and the correct asymptotic behaviour as $m_q \rightarrow \infty$ to extrapolate the neutron and proton magnetic moments

$$\mu_i(m_\pi) = \frac{\mu_0}{1 - \frac{\chi_i}{\mu_0} m_\pi + c m_\pi^2}. \quad (4)$$

The chiral coefficients for the isovector and isoscalar moments are $\chi_v = -8.82$ and $\chi_s = 0$ respectively and μ_0 and c are fitting parameters, to be determined by the lattice data.

In order to build a suitable extrapolating function for the Q^2 -dependence of the form factors, we use the connection between the mass parameter in a dipole form factor and the corresponding mean-square radius. For the isovector magnetic form factor the mean square radius is

$$\langle r^2 \rangle_M^v = -\frac{6}{G_M^v(0)} \frac{dG_M^v}{dQ^2} \Big|_{Q^2=0}. \quad (5)$$

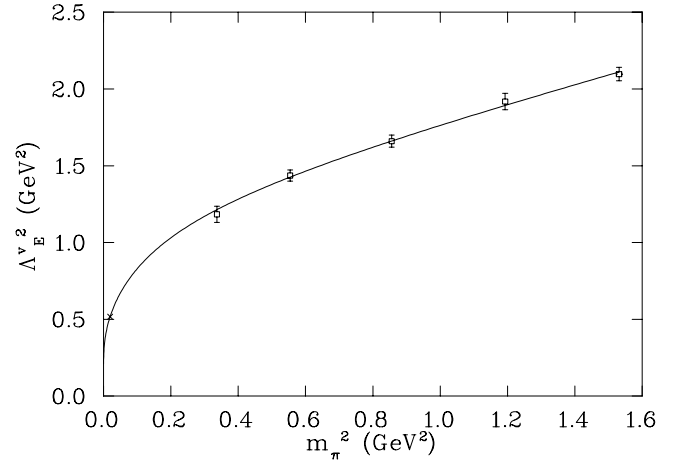


Fig. 1. Fit to values of the isovector electric form factor dipole mass extracted from lattice data with lattice spacing $a = 0.051$ fm. The physical value predicted by the fit is also indicated

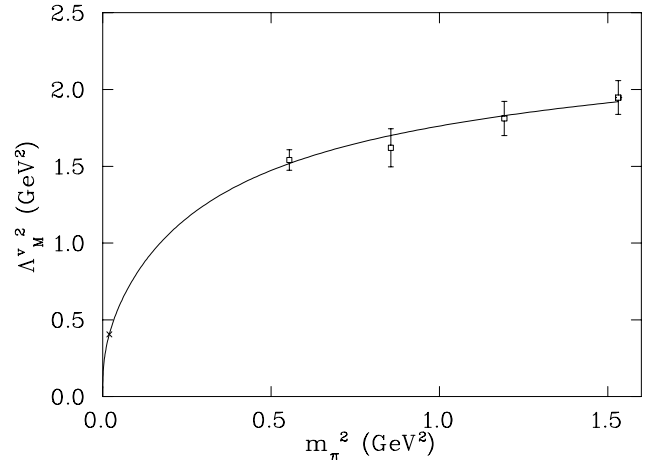


Fig. 2. Fit to values of the isovector magnetic form factor dipole mass extracted from lattice data with lattice spacing $a = 0.051$ fm. The physical value predicted by the fit is also indicated

Comparing this to the dipole of (1), we find an expression relating the dipole mass to $\langle r^2 \rangle_M^v$,

$$(\Lambda_M^v)^2 = \frac{12}{\langle r^2 \rangle_M^v}. \quad (6)$$

The chiral behaviour of the magnetic mean squared radius is known from chiral perturbation theory [30]

$$\langle r^2 \rangle_M^v \sim \frac{\chi_1}{m_\pi} + \chi_2 \ln\left(\frac{m_\pi}{\mu}\right). \quad (7)$$

The constants χ_1 and χ_2 are given by

$$\chi_1 = \frac{g_A^2 m_N}{8\pi f_\pi^2 \kappa_v}, \quad (8)$$

$$\chi_2 = -\frac{5g_A^2 + 1}{8\pi^2 f_\pi^2}, \quad (9)$$

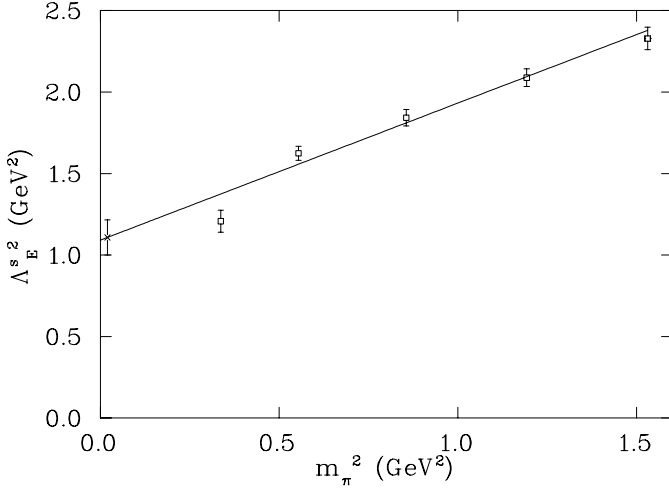


Fig. 3. Linear fit to values of the isoscalar electric form factor dipole mass extracted from lattice data with lattice spacing $a = 0.051$ fm. The physical value predicted by the fit is also indicated

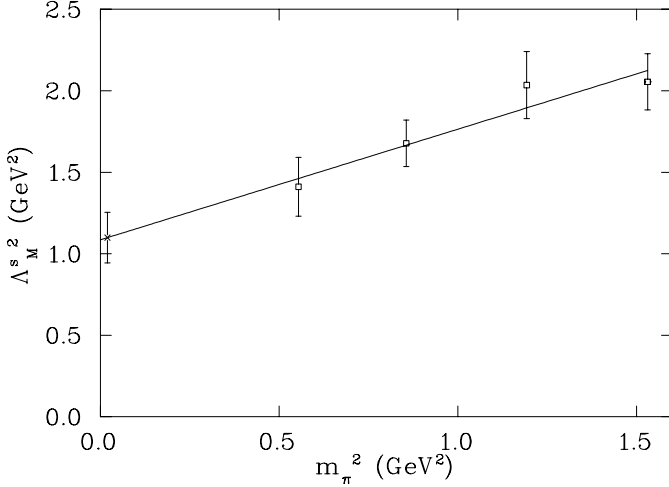


Fig. 4. Linear fit to values of the isoscalar magnetic form factor dipole mass extracted from lattice data with lattice spacing $a = 0.051$ fm. The physical value predicted by the fit is also indicated

where $g_A = 1.27$ is the axial coupling constant and $f_\pi = 93$ MeV is the pion decay constant, $m_N = 940$ MeV is the nucleon mass and $\kappa_v \approx 4.2$ is the isovector anomalous magnetic moment of the nucleon (in the chiral limit).

Earlier experience with nucleon properties as a function of quark mass has suggested that, as a consequence of the finite size of the source of the pion field, chiral loops are strongly suppressed for $m_\pi > 0.4$ GeV [31]. Thus in order to build an effective chiral extrapolation formula one needs to modify the non-analytic terms given above so that they are suppressed above this mass. For guidance as to an appropriate parametrization of this suppression we have evaluated the pion loops that give rise to the non-analytic chiral terms using the Cloudy Bag Model (CBM) [32,33,

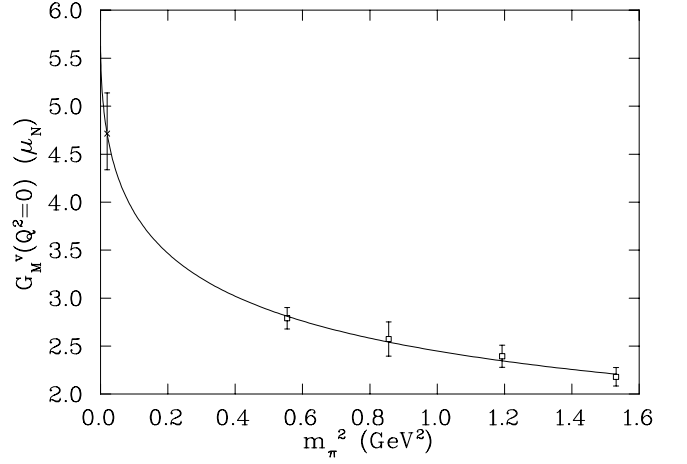


Fig. 5. Fit to values of the isovector magnetic moment extracted from lattice data with lattice spacing $a = 0.051$ fm. The physical value predicted by the fit is also indicated

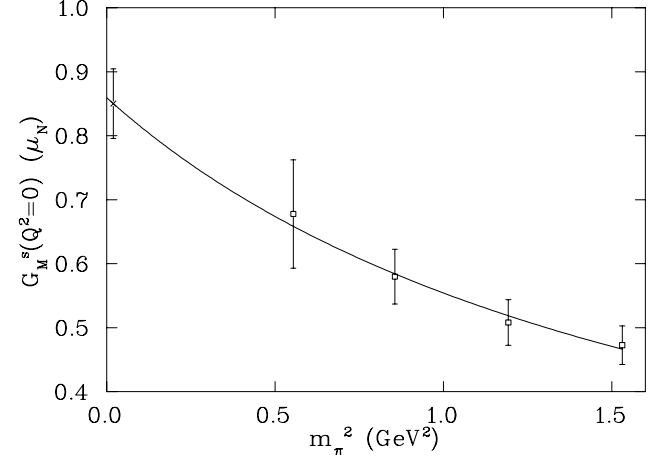


Fig. 6. Fit to values of the isoscalar magnetic moment extracted from lattice data with lattice spacing $a = 0.051$ fm. The physical value predicted by the fit is also indicated

34]. The results led us to replace (7) with the expression

$$\langle r^2 \rangle_M^v \sim \frac{\chi_1}{m_\pi} \frac{2}{\pi} \arctan(\mu/m_\pi) + \frac{\chi_2}{2} \ln\left(\frac{m_\pi^2}{m_\pi^2 + \mu^2}\right), \quad (10)$$

which ensures the correct chiral behaviour at low- m_π but suppresses it at values larger than μ .

Substituting this functional form into (6) and introducing a linear dependence on m_π^2 to give the expected behaviour at large quark mass, we get the following expression for the dipole mass associated with the isovector magnetic form factor

$$(\Lambda_M^v)^2 = \frac{12(1 + A_1 m_\pi^2)}{A_0 + \frac{\chi_1}{m_\pi} \frac{2}{\pi} \arctan(\mu/m_\pi) + \frac{\chi_2}{2} \ln\left(\frac{m_\pi^2}{m_\pi^2 + \mu^2}\right)}. \quad (11)$$

Here A_0 and A_1 are unknown parameters, adjusted to fit the lattice data.

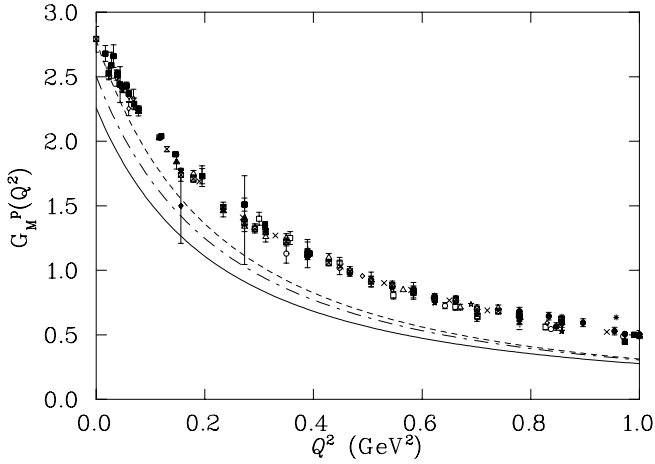


Fig. 7. The magnetic form factor for the proton extrapolated from lattice data with lattice spacings $a = 0.093$ fm (single line), $a = 0.068$ fm (dash-dot line) and $a = 0.051$ fm (dotted line) and compared to experimental results

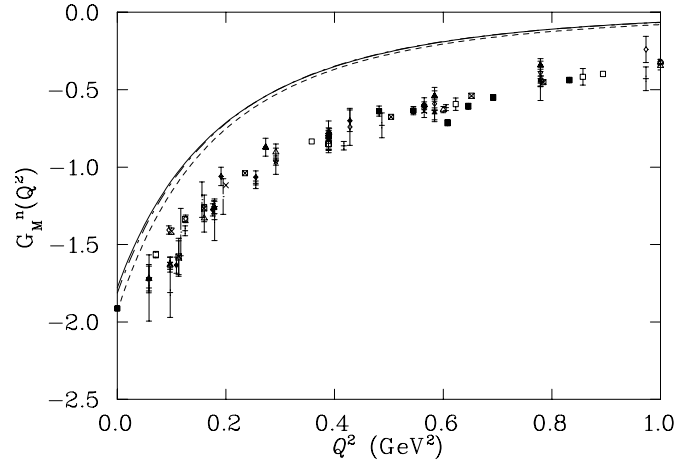


Fig. 8. The magnetic form factor for the neutron extrapolated from lattice data with lattice spacings $a = 0.093$ fm (single line), $a = 0.068$ fm (dash-dot line) and $a = 0.051$ fm (dotted line) and compared to experimental results

We can perform a similar analysis to find an expression for the isovector electric form factor dipole mass Λ_E^v , combining the non-analytic chiral behaviour predicted by χ PT at low m_π^2 with the slowly varying behaviour observed in lattice QCD at large masses. Using the equivalent of (5) for the electric form factor, we can relate the dipole mass to the mean charge radius by

$$(\Lambda_E^v)^2 = \frac{12}{\langle r^2 \rangle_E^v}. \quad (12)$$

The chiral behaviour of the isovector mean charge radius is like that of the magnetic radius but with no $1/m_\pi$ term [30],

$$\langle r^2 \rangle_E^v \sim \chi_2 \ln\left(\frac{m_\pi}{\mu}\right). \quad (13)$$

Combining this chiral behavior with a linear dependence on m_π^2 produces the following predicted form for the isovector electric form factor dipole mass,

$$(\Lambda_E^v)^2 = \frac{12(1 + B_1 m_\pi^2)}{B_0 + \frac{\chi_2}{2} \ln\left(\frac{m_\pi^2}{m_\pi^2 + \mu^2}\right)}, \quad (14)$$

where B_0 and B_1 are general fitting parameters.

Having isolated the chiral non-analytic behaviour to the isovector form factors, we expect that the isoscalar dipole masses should be roughly linear in m_π^2 – as observed in CBM results.

The forms chosen to represent the dipole masses as a function of m_π are able to reproduce the predictions of the CBM [34] very well.

3 Results

The QCDSF Collaboration [7] has recently reported QQCD results for the isovector and isoscalar nucleon electric and magnetic form factors calculated at different Q^2

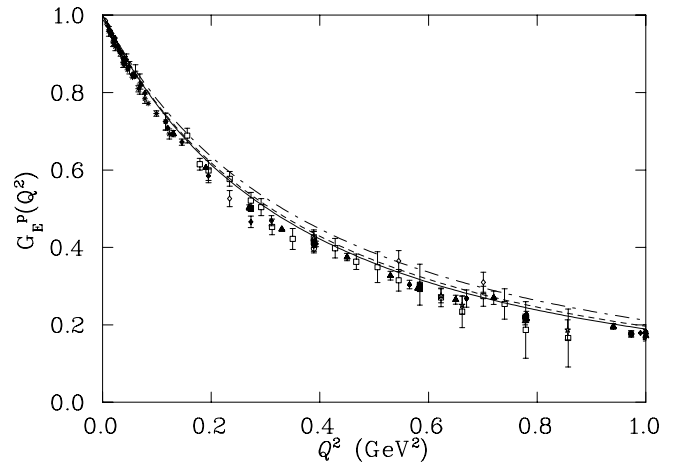


Fig. 9. The electric form factor for the proton extrapolated from lattice data with lattice spacings $a = 0.093$ fm (single line), $a = 0.068$ fm (dash-dot line) and $a = 0.051$ fm (dotted line) and compared to experimental results

values and pion masses and for three different lattice spacings ($\beta = 6.0, 6.2$ and 6.4). Using this data, with the scale set using the Sommer method ($r_0 = 0.5$ fm [35]), we have plotted the form factors G_M^v , G_M^s , G_E^v and G_E^s , calculated on the lattice as a function of Q^2 , at each pion mass and lattice spacing, and fit each graph with a dipole form, finding a best-fit dipole mass and magnetic moment. Plots of these best-fit dipole masses and magnetic moments against m_π^2 for the lattice spacing $a = 0.051$ fm are shown in Figs. 1, 2, 3 and 4.

The electric and magnetic isovector dipole masses are fitted with functions of the form (11) and (14), respectively. The scale $\mu = 0.41$ GeV was chosen to give the best simultaneous fit to the lattice data for both the electric and magnetic isovector form factors at all three lattice spacings. We observe that the curves fit the data well and

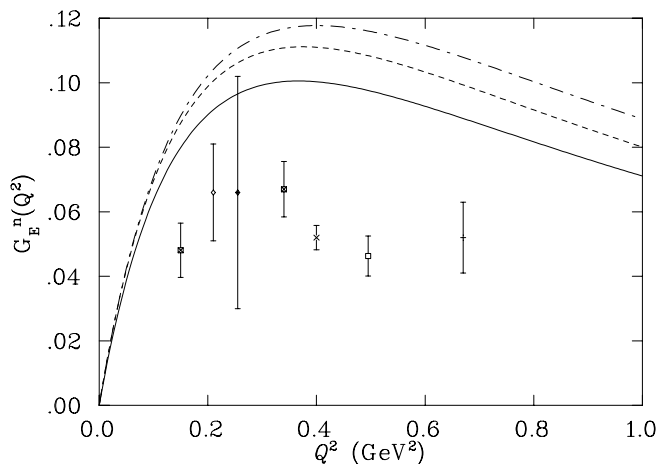


Fig. 10. The electric form factor for the neutron extrapolated from lattice data with lattice spacings $a = 0.093$ fm (single line), $a = 0.068$ fm (dash-dot line) and $a = 0.051$ fm (dotted line) compared to experimental results

predict physical results significantly different from those obtained by a naive linear extrapolation. In both cases, the form of the fitting functions greatly restricts the physical value, resulting in a small error. Finally, we note that the electric and magnetic isoscalar dipole mass plots are fitted with linear functions in m_π^2 , because of the absence of any leading non-analytic behaviour in this case.

The magnetic moment plots are extrapolated using the Padé approximant, (4). The resulting errors in the physical magnetic moments dominate over errors in the dipole mass and are the main source of error in our results for the physical magnetic form factors. Similar fits were produced for lattice results at lattice spacings $a = 0.068$ fm and $a = 0.093$ fm.

Using the extrapolated physical values for the dipole masses and magnetic moments, we reconstruct the electric and magnetic form factors for the proton and neutron as a sum of two dipoles in Q^2 . Figures 7, 8, 9 and 10 show the electric and magnetic form factors for the proton and neutron extrapolated from that lattice data at each of the three lattice spacings ($a = 0.093$ fm, $a = 0.068$ fm and $a = 0.051$ fm) compared to the experimental data from [36].

Except for the case of the neutron electric form factor (which is very sensitive because it is the result of a cancellation between G_E^v and G_E^s which yields a relatively small result) our extrapolated curves match the experimental points very well, particularly for G_E^p and G_M^p . Although our curve for G_E^n is almost twice as big as the experimental values, it does describe the correct shape but peaks at slightly too high a value of Q^2 .

4 Conclusion

We have proposed a relatively simple approach to the extrapolation of lattice QCD data for the nucleon electromagnetic form factors. The data from QCDSF has been

parametrized by a simple dipole form, with the dipole mass parameter taken to be a function of m_π which incorporates the leading non-analytic behaviour of chiral perturbation theory. For the isoscalar case, where there is no leading non-analytic behaviour, the extrapolation is a simple linear function of m_π^2 , while for the isovector case we have used a functional form suggested by studies based on the cloudy bag model, which suppress the chiral behaviour for pion masses larger than 400 MeV.

The level of agreement between the empirical Q^2 dependence of the proton electric form factor and the proton and neutron magnetic form factors is impressive. This is quite remarkable when one considers that the data is based on quenched approximation and is extrapolated from a light quark mass a factor of twenty above the physical mass. This is only possible because of the remarkable fact that pion loops are suppressed by finite size effects once the pion Compton wavelength is smaller than the size of the source – empirically, $m_\pi > 0.4$ GeV. The extrapolation of the data has, of course, been performed using chiral coefficients appropriate to full QCD.

One of the main open questions in the analysis concerns the neutron electric form factor, which will always be a challenge given that it vanishes at $Q^2 = 0$ – as a result of the cancellation between the isoscalar and isovector contributions. In addition, we observe that there appears to be some residual β dependence in the value of the nucleon magnetic moments and this is the major ambiguity in the current reconstruction of the proton and neutron magnetic form factors. It is encouraging that the results for $\beta = 6.4$, corresponding to the smallest lattice spacing, is in the best agreement with the experimental data. Future lattice simulations will undoubtedly clarify this issue and also give us data at lower values of the quark mass. It would also be valuable to have full QCD simulations (rather than QQCD) to ensure that there is no unanticipated systematic difference. Most importantly, as the accuracy of the data and the range of Q^2 covered increases, we can extend the present analysis by employing more complicated fitting functions which should allow us to test the behaviour of properties such as G_E/G_M against experiment.

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